

B.Sc Part-II.

Paper — II

Wein's displacement law

This law states that the product of the wavelength corresponding to maximum energy $\lambda_m T = \text{constant}$.

This constant is called Wein's displacement constant. And has value $0.2896 \times 10^{-12} \text{ mK}$. According to this law λ_m decreases with increase in temperature.

Deduction : — Let us imagine a spherical enclosure of perfectly reflecting walls and capable of expanding. Let it be filled with diffuse radiations of energy density U at a uniform temperature T . If V is the volume of the enclosure, then total internal energy of radiation is given by

$$U = uV$$

Let us now suppose that the walls of the enclosure move outward slowly with uniform velocity so that the radiation inside it expands adiabatically. If dV is the change in volume of the enclosure, the work done by the pressure of radiation on the walls of the enclosure = PdV which is drawn from internal energy of radiation. If dU is the decrease in internal energy, then from the first law of thermodynamics, we have

$$dU + PdV = dQ = 0 \rightarrow ①$$

(\because change is adiabatic, $dQ=0$)

Also $P = \frac{u}{3}$. hence Substituting the values of P and U in equation ①.

$$\text{we get, } d(uV) + \frac{1}{3}udV = 0$$

$$\text{or, } udV + Vdu + \frac{1}{3}udV = 0$$

$$\text{or, } \frac{4}{3}udV + Vdu = 0$$

(P.T.O)

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$$\text{or, } \frac{4}{3} u dv = -v du$$

$$\text{or } \frac{4dv}{3v} = -\frac{du}{u}$$

$$\text{or } \frac{4}{3} \int \frac{dv}{v} = - \int \frac{du}{u} \rightarrow ②$$

$$\text{or, } \frac{4}{3} \log v = - \log u + \text{constant.}$$

$$\text{or, } \frac{4}{3} \log u = \text{constant.}$$

$$\text{or, } \log v^{4/3} \cancel{\text{at } u} = \text{constant.}$$

$$\text{But, } u = AT^4, \text{ where } A = \text{constant.}$$

$$\therefore \log(v^{4/3} AT^4) = \text{constant.}$$

$$\text{or, } \log(v^{4/3} T^4) + \cancel{\text{constant}} = \log k$$

$$\therefore v^{4/3} T^4 = \text{constant.} \rightarrow ③$$

$$\text{or } v^{4/3} \propto \frac{1}{T^4}$$

$$\text{since, we have the equation, } \frac{\partial \lambda}{\lambda} = \frac{\partial r}{r} \rightarrow ④$$

we know that volume of sphere $V = \left(\frac{4}{3}\right)\pi r^3$ so that the change in volume of sphere

$$\partial V = \left(\frac{4}{3}\right)\pi \cdot 3r^2 \partial r$$

$$\therefore \frac{\partial r}{r} = \frac{1}{3} \frac{\partial V}{V} \rightarrow ⑤$$

$$\therefore \text{from equation } ④ \text{ we get, } \int \frac{\partial \lambda}{\lambda} = \frac{1}{3} \int \frac{\partial V}{V} \rightarrow ⑥$$

$$\text{Solving above equation, we have, } \lambda = K V^{1/3}$$

$$\therefore V^{1/3} = \lambda/K \rightarrow ⑦$$

$$\therefore \text{from equation } ③, \text{ we get, } \frac{\lambda}{K} \cdot T = \text{constant.}$$

$$\text{or } \lambda T = \text{constant} \rightarrow ⑧$$

this is Weeme displacement law.